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During this contract, the research has focused on a related but different topic: analysis of nonlinear and/or time-varying systems using quadratic Lyapunov functions and convex optimization. Most importantly, these methods extend to state feedback synthesis for nonlinear, time-varying, and uncertain systems, and so should prove extremely useful in many industrial control problems.

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Final Technical Report
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Numerical Methods for Determining Performance Limits of Linear Control

Principal Investigator: Stephen P. Boyd, Stanford University

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Research Objectives and Background

The basic question addressed is: how can we use the enormous computing power now available to do better control system design and analysis? Of course, we can use the increase in computing power to solve larger problems, or to solve the same problems faster. However, the question we address is: are there completely new methods or techniques that are possible because of the computer power?

A major theme in the research is convex optimization. Roughly speaking, convex optimization problems are tractable, even when they are large and nondifferentiable. (This statement can be argued on a theoretical or practical level.) Moreover, over the last decade, researchers have made significant advances in algorithms for convex optimization. The best known is the development of powerful interior point optimization methods. Other research suggests that algorithms for convex problem are ideally suited for implementation on distributed computers.

The basic approach is to identify problems in control systems design and analysis that can be cast (or re-cast) as convex optimization problems. When this is done for a particular class of problems, we can make several conclusions: first, the fundamental computational complexity of the problem is low; second, from a practical point of view we can consider the problem "solved" since very efficient and totally effective numerical methods can be used to reliably compute the solution. Since the class of convex problems is much larger than the class of problems for which an analytical solution is known, this basic approach allows the engineer to consider problems that are much closer to the real engineering problem than would be possible using a purely "analytical" approach.

Up until 1991, most of the research concentrated on formulating linear controller design problems as convex optimization problems. This topic is investigated in a series of papers from 1986 to 1992, and a book published in 1991 (all with AFOSR support). Although the book is primarily an advanced research monograph, it is already into the second printing (3500 copies to date), and has been read by essentially all researchers in control theory, and many industrial and commercial engineers. A simple code called QDES was developed in 1986-7 to demonstrate the the use of these ideas in computer aided control system design, and is still used by several sites now (although it was never meant to serve any purpose other than to demonstrate the feasibility of using a computer to synthesize a controller directly from functional specifications). The basic ideas of the research have been used in several successful commercial applications, ranging from control of a rapid thermal processor (in collaboration with Texas Instruments and DARPA) to process design and control optimization of a Fiber Pyrolysis Process (3M and Integrated Systems Inc). The research on RTP has attracted wide attention in industry; already several companies involved in the manufacture of RTP equipment have decided to change future designs to take advantage of these methods; within several years the methods his group pioneered will be the industry standards. Integrated Systems Inc will be developing commercial computer aided control design software tools based on Professor Boyd's convex optimization approach.

Research Summary

During this contract, the research has focussed on a related but different topic: analysis of nonlinear and/or time-varying systems using quadratic Lyapunov functions and convex optimization. Most importantly, these methods extend to state feedback synthesis for nonlinear, time-varying, and uncertain systems, and so should prove extremely useful in many industrial control problems.

In recent research we have shown that a wide variety of problems in systems and control theory can be cast or recast as convex problems that involve linear matrix inequalities (LMIs). For a few very special cases there are "analytical solutions" to these problems, but in general they can be solved numerically very efficiently. In many cases the inequalities have the form of simultaneous Lyapunov or algebraic Riccati inequalities; such problems can be solved in a time that is comparable to the time required to solve the same number of Lyapunov or Algebraic Riccati equations. Therefore the computational cost of extending current control theory that is based on the solution of algebraic Riccati equations to a theory based on the solution of (multiple, simultaneous) Lyapunov or Riccati inequalities is modest.

Examples include: multicriterion LQG, synthesis of linear state feedback for multiple or nonlinear plants ("multi-model control"), optimal transfer matrix realization, norm scaling, synthesis of multipliers for Popov-like analysis of systems with unknown gains, and many others. Full details are in the references listed below.

In the last year, we continued and expanded our effort in matrix inequalities in system and control theory. The *main achievement* was the publication of the book [BEFB94], which sets forth the basic ideas, and shows how approximately one hundred system and control problems can be cast as convex optimization problems involving matrix inequalities. While many of the results appearing in the book had already been published, many had not. The book has attracted interest in both control engineering and, surprisingly, optimization. Several factors, along with the publication of this book, have combined to give this new approach momentum: the appearance of Gahinet's commercial toolbox LMI-Lab (which gives researchers and engineers the ability to try the methods immediately); new results in super-efficient computational algorithms [BVG94, VB95]; new results from researchers such as Safonov, Packard, and others extending the methods to structured controllers, gain scheduling and parameter-dependent controllers, and so on. Finally, new and startling results in combinatorial optimization, that at first sight might not appear related, have been shown to be closely connected [VB94b] (see below).

We had much progress in the algorithm/software/tool development area. Vandenberghe and Boyd designed and implemented a complete set of software for semidefinite programming (SDP), called SP [VB94c]. This code consists of a user manual and reference, complete C source, Matlab interfaces, and many examples. It exploits block diagonal structure, and is based on LAPACK, for which efficient libraries are widely available. The code complements a survey paper written for SIAM Review (see below). It is distributed via anonymous ftp,

and is currently in use at many university and research groups, and several companies.

Boyd and Wu conceived, designed, and implemented the code SDPSOL, a parser/solver for semidefinite programs with matrix structure [BW95]. We developed a language in which the SDPs that arise in control are easily described in a natural form. SDPSOL parses such specifications, and solves them using a new primal-dual interior-point method. SDPSOL has already been used at Stanford for several projects, including control of semiconductor manufacturing equipment, problems in cellular communications, combinatorial optimization, and information theory. We are currently distributing an alpha version, to get feedback from users on the language design. The current version does not exploit the problem structure in the computations, and so should only be considered a 'pilot' for evaluating the concept and language. The next version will incorporate problem structure into the computational engine.

We have also continued our effort at developing super-efficient algorithms. We have made some preliminary investigations into the use of sparse-matrix methods for SDP. The results so far have been mixed; we have not reported these results yet. The potential payoff, however, is great: relatively standard code that solves even very large SDPs and LMIs from control at speeds comparable to LP codes.

In the area of fundamental theory, we have made one important contribution over the last year — a detailed discussion of the connection between the uses of SDP in control and combinatorial optimization. We wrote the survey paper *Semidefinite Programming* for SIAM Review [VB94b] (see also [VB94a]), emphasizing the connections between control and other applications. SDP arises in many fields of engineering, but is currently being most intensively studied in control and combinatorial optimization. At first glance these two applications do not appear to be related, but a deeper look reveals that they are quite similar. In fact the S-procedure (and its relatives), used in control since 1944, is equivalent to the SDP relaxations of combinatorial problems that are now attracting such great attention. New results form combinatorial optimization, which give uniform bounds on how suboptimal the approximations can be, offer hope that we can prove something about SDP/LMI relaxations of hard control problems, i.e., BMIs. One nontechnical but important consequence of our work is that the LMIs and SDPs arising in control are now being studied by the very best and brightest in the field of optimization — Todd, Overton, Wolkowicz, Nesterov, Nemirovsky and others.

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